

محلہ کرتے ہیں : نمونہ سوال اٹھائی

شمارہ سوال

$$x^n y^m = (x+y)^{n+m}$$

$$n x^{n-1} y^m + m x^n y^{m-1} y' = (n+m) (x+y)^{n+m-1} (1+y')$$

$$n x^{n+1} y^{m+1} + m x^{n+2} y^m y' = (n+m) x^2 y (x+y)^{n+m-1} (1+y')$$

$$\text{چونکہ } xy(x+y) \Rightarrow n x^n y^{m+n} (x+y) + m x^{n+1} y^m (x+y) y' = (n+m) xy(x+y)^n (1+y')$$

$$\Rightarrow n y(x+y) + m x(x+y) y' = (n+m) (xy) (1+y')$$

$$(m x(x+y) - (n+m) xy) y' = (n+m) (xy) - n y(x+y)$$

$$(m x^2 - n xy) y' = m xy - n y^2$$

$$x(m x - n y) y' = y(m x - n y) \Rightarrow$$

$$x y' = y$$

$$\begin{aligned} \sin(x+a) &= \sin x \cos a + \cos x \sin a \quad \#96 \\ \cos(x+a) &= \cos x \cos a + (-\sin x) \sin a \quad \text{P.144} \\ \cos(x+a) &= \cos x \cos a - \sin x \sin a \\ x^2 - 2x - 8 &= 0 \Rightarrow 2x - 2 = 0 \Rightarrow x = 1 \end{aligned}$$

$$\begin{aligned} \frac{(1+\sqrt{3}i)^6}{(2+2i)^4} \\ |1+\sqrt{3}i| &= \sqrt{1+3} = 2, \quad \tan^{-1}\theta = \frac{\sqrt{3}}{1} \Rightarrow \theta = \frac{\pi}{3} \\ 1+\sqrt{3}i &= 2(\cos \pi/3 + i \sin \pi/3) = 2e^{i\pi/3} \\ |2+2i| &= \sqrt{4+4} = 2\sqrt{2}, \quad \tan^{-1}\theta = 1 \Rightarrow \theta = \pi/4 \\ \Rightarrow 2+2i &= 2\sqrt{2}(\cos \pi/4 + i \sin \pi/4) = 2\sqrt{2}e^{i\pi/4} \\ \frac{(1+\sqrt{3}i)^6}{(2+2i)^4} &= \frac{(2e^{i\pi/3})^6}{(2\sqrt{2}e^{i\pi/4})^4} = 2 \frac{e^{2\pi i}}{e^{\pi i}} = 2e^{-\pi i} = -2 \end{aligned}$$

$$\begin{aligned} \sqrt[4]{-2} &= ? \quad , \quad -2 = 2 \cos \pi + i \sin \pi = 2e^{\pi i} \\ z &= \sqrt[4]{-2} = \sqrt[4]{2} e^{\frac{(\pi + 2k\pi)}{4}i}, \quad k=0,1,2,3 \\ z_0 &= \sqrt[4]{2} e^{\pi/4 i} = \sqrt[4]{2} (\sqrt{2}/2 + i\sqrt{2}/2) \\ z_1 &= \sqrt[4]{2} (\cos \frac{3\pi}{4} + i \sin \frac{3\pi}{4}) = \sqrt[4]{2} (-\sqrt{2}/2 + i\sqrt{2}/2) \\ z_2 &= \sqrt[4]{2} (-\cos \pi/4 - i \sin \pi/4) = -\sqrt[4]{2} (\sqrt{2}/2 + i\sqrt{2}/2) \\ z_3 &= \sqrt[4]{2} (\cos 7\pi/4 + i \sin 7\pi/4) = \sqrt[4]{2} (\sqrt{2}/2 - i\sqrt{2}/2) \end{aligned}$$

$$f(x) = \begin{cases} 1/x & |x| > c \\ a+bx^2 & |x| \leq c \end{cases}, \quad f'(c) = ?$$

$$f'_-(c) = \lim_{x \rightarrow c^-} \frac{f(x) - f(c)}{x - c} = \lim_{x \rightarrow c^-} \frac{a+bx^2 - (a+bc^2)}{x - c}$$

$$= \lim_{x \rightarrow c^-} \frac{b(x-c)(x+c)}{x-c} = 2bc$$

$$f'_+(c) = \lim_{x \rightarrow c^+} \frac{f(x) - f(c)}{x - c} = \lim_{x \rightarrow c^+} \frac{1/x - (a+bc^2)}{x - c}$$

$$= \lim_{x \rightarrow c^+} \frac{\frac{1}{x} - (a+bc^2)}{x - c} = \lim_{x \rightarrow c^+} \frac{1 - (a+bc^2)x}{x(x-c)}$$

$$\lim_{x \rightarrow c^+} \frac{1 - (a+bc^2)x}{x(x-c)} = 2bc$$

$$\lim_{x \rightarrow c^+} f(x) = \lim_{x \rightarrow c^+} \frac{1}{x} = \frac{1}{c}, \quad \lim_{x \rightarrow c^-} f(x) = \lim_{x \rightarrow c^-} (a+bx^2) = a+bc^2$$

$$\Rightarrow \frac{1}{c} = a+bc^2 \Rightarrow a+bc^2 = \frac{1}{c}$$

$$= \lim_{x \rightarrow c^+} \frac{(a+bc^2)c - (a+bc^2)x}{x(x-c)} = \lim_{x \rightarrow c^+} \frac{-(a+bc^2)(x-c)}{x(x-c)}$$

$$= -\frac{a+bc^2}{c} = 2bc \Rightarrow a+bc^2 = 2bc^2$$

$$\Rightarrow a - bc^2 = 0$$

$$\begin{cases} a+bc^2 = \frac{1}{c} \\ a-bc^2 = 0 \end{cases} \Rightarrow 2a = \frac{1}{c} \Rightarrow a = \frac{1}{2c}$$

$$\frac{1}{2c} - bc^2 = 0 \Rightarrow b = \frac{1}{2c^3}$$

سؤال #111
p.146

$$f(x) = \begin{cases} x^2 \sin \frac{1}{x} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$$

شماره سؤال

1) $x \neq 0, f'(x) = 2x \sin \frac{1}{x} - \frac{1}{x^2} x^2 \cos \frac{1}{x}$
 $= 2x \sin \frac{1}{x} - \cos \frac{1}{x}$

2)

$$\lim_{x \rightarrow 0} f(x) = \lim_{x \rightarrow 0} x^2 \sin \frac{1}{x} = 0 = f(0)$$

$$-1 \leq \sin \frac{1}{x} \leq 1 \Rightarrow -x^2 \leq x^2 \sin \frac{1}{x} \leq x^2$$

$$\Rightarrow \lim_{x \rightarrow 0} x^2 \sin \frac{1}{x} = 0$$

3) $x=0$ منقذ نیست؟

$$f'(0) = \lim_{x \rightarrow 0} \frac{f(x) - f(0)}{x - 0} = \lim_{x \rightarrow 0} \frac{x^2 \sin \frac{1}{x} - 0}{x}$$

$$= \lim_{x \rightarrow 0} x \sin \frac{1}{x} = 0$$

4) $x=0$ منقذ نیست؟

$$\lim_{x \rightarrow 0} f'(x) = \lim_{x \rightarrow 0} (2x \sin \frac{1}{x} - \cos \frac{1}{x})$$

این حد در $x=0$ وجود ندارد و $\lim_{x \rightarrow 0} \cos \frac{1}{x}$ وجود ندارد. f' در $x=0$ منقذ نیست.